

SPOUTING CHARACTERISTICS OF SMALL GLASS PARTICLES WITH WATER

Seung Jai Kim

Department of Chemical Engineering, Chonnam National University, Kwangju, 505

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Abstract—Experimental data are reported for the minimum spouting velocity, bed pressure drop, maximum spoutable height and spout diameter which show that spouting of small particles with water represents a new spouting regime. The transition from large to small particle spouting occurs when the voidage at the inlet to the spout goes from unity to less than unity.

INTRODUCTION

Spouting is a technique for bringing a fluid into contact with solid particles in which the fluid is introduced vertically as a jet into a bed of particles. Specific applications of spouted bed technology are discussed by Mathur et al. [1, 2]. The fluid entering the bed produces a central spout containing particles which move upward with the fluid. The entrained solids in the spout are carried upward and after reaching the top of the bed fall back into the top of the annulus. The particles in the annulus move downward and reenter the spout establishing a systematic particle circulation in the bed. The fluid leaks from the spout into the annulus across the spout-annulus interface.

There are several parameters for describing the spouting for a given fluid-particle system. These basic parameters are the minimum spouting velocity, pressure drop at minimum spouting, maximum spoutable height and spout diameter.

Many expressions have been reported to predict these parameters for coarse particles [4-12], however, the study for fine particles are limited.

The small particle systems are of interest theoretically because the inertial force of the jet at minimum spouting is generally small relative to the bed pressure drop. The A parameter [10] which is the measure of the ratio of those forces has been shown to be important in predicting the maximum spoutable height and spout diameter in beds of coarse spherical particles when A is greater than about 0.02. A dimensionless plot for predicting the maximum spoutable height shows that there is a break in the curve when A is about 0.02 [9, 10].

Since the equations for predicting the basic spouting parameters for fine particles of A less than 0.02 are not

available, the data for those parameters are obtained, correlated and compared with those for coarse particles. In addition, the spout voidage was calculated from the momentum equation and spouting regime was discussed.

EXPERIMENTS

Equipment and Measurement of the bed Properties

The experiments were carried out in the apparatus shown schematically in Figure 1. The spouted bed shown in the figure is a cylindrical half column, 50.8 mm in diameter and 950 mm high. It is made of Plexiglas, has a flat base and was designed in such a way that semi-circular inlet tubes of different diameters could be attached to the column. The flat surface of the half column has a non-uniform rectangular grid of 43 pressure taps all connected to piezometer tubes. The axial distance between the taps was smallest close to the spout inlet in order to obtain a good axial pressure profile there. The taps in the annulus were used in experiments concerned with the annular flowfield. The flowrates were measured using rotameters and the water temperature was generally kept at about 23°C.

Special precautions were provided to remove dissolved air from the water by heating it to about 50°C and then cooling. This water was stored in a 0.2 m³ tank from which it was pumped to the bed. As a result of this deaerating procedure, no bubbles were observed anywhere in the bed.

A 101.6 mm diameter circular column made of Plexiglas was used to carry out fluidization experiments in order to determine u_{mf} , ε_{mf} and ΔP_{mf} . The mean diameter and sphericity of particle were determined.

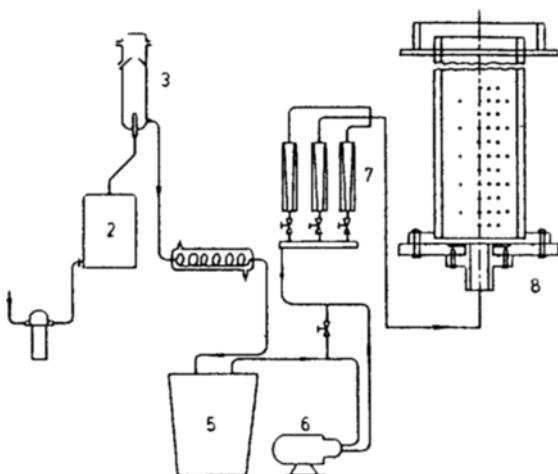


Fig. 1. Schematic diagram of experimental apparatus: 1: water filter, 2: water heater, 3: air separator, 4: water cooler, 5: water tank, 6: pump, 7: rotameter, 8: spouted bed; arrows indicate the direction of water flow.

from photographs assuming that the actual particle shape could be modeled as an ellipsoid.

Determination of u_{ms} , ΔP_{ms} , H_m and d_s

The bed pressure drop, annulus height, jet penetration and spout fountain height are shown in Figure 2 as a function of flowrate for 0.995 mm particles. The bed is shallow enough for the jet to penetrate through it. With increasing flowrate in this system, the jet begins to penetrate the bed at a flowrate below V_{mf} and slightly before the peak pressure is reached. As the flowrate is increased, penetration continues rapidly with decreasing bed pressure drop until the jet reaches the top of the bed at point A'. On decreasing the flowrate from above the minimum spouting, the jet ceases to penetrate the bed at point A. This flowrate, as seen in Figure 2, is defined as the minimum spouting flowrate. $(V_{ms})_{Hm} > V_{mf}$

primarily due to wall effects as the spouted bed is a 50.8 mm half-column ($D_H = 15.52$ mm) and the fluidizing column has a 101.6 mm diameter ($D_H = 25.4$ mm). ΔP_{ms} is the experimental value of the spouting pressure drop, $[p_s(0) - p_s(H)]$, measured at u_{ms} obtained by extrapolating the measured pressure along the axis of symmetry to $z = 0$. A pressure tap at $z = 0$ just above the screen covering the spout inlet tube gives the same value for ΔP_{ms} a the extrapolation procedure.

Above a certain bed height, the jet does not penetrate to the top of the bed regardless of the flowrate because the bed expands faster than the jet can penetrate it.

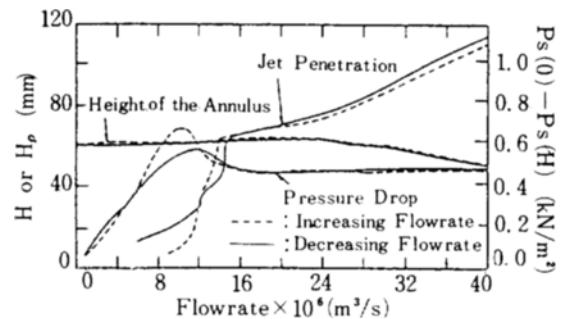


Fig. 2. Pressure drop, annular bed height, jet penetration and spout fountain height as a function of flowrate; $d_s = 0.995$ mm, $d_i = 6.35$ mm, $\Delta P_{ms} = 0.607$ kN/m².

rate it. Above the velocity, $(u_{ms})_{Hm}$, the bed consists of two distinct zones: a lower spouted bed zone and an upper fluidized bed zone. When the bed height is less than H_m , only the lower spouted bed zone exists [13].

The criterion for determining H_m and $(u_{ms})_{Hm}$ on these beds was formulated by observation of the change in the shape of the jet with decreasing flowrate. As the flowrate is lowered from above $(V_{ms})_{Hm}$, the jet narrows at the top finally assuming a flame tip shape. $(V_{ms})_{Hm}$ is the highest flowrate for which the flame tip shape is visible and the height of the jet to the tip of the flame at $(V_{ms})_{Hm}$ is denoted H_m . $(\Delta P_{ms})_{Hm}$ is the pressure drop at $(V_{ms})_{Hm}$.

The general behavior of the bed is unchanged by changing d_i . With the largest spout inlet tube (19.05 mm in diameter), the jet penetrates rapidly with increasing flowrate, as before, once the peak pressure is reached in the bed. On decreasing the flowrate, the jet ceases to penetrate to the top of the bed at a flowrate above V_{mf} so that again $(V_{ms})_{Hm} > V_{mf}$. However, in shallow beds V_{mf} will exceed V_{ms} . ΔP_{ms} is as before determined by the extrapolation procedure and two distinct zones are again observed in beds deeper than H_m .

RESULTS

Bed Properties of the Fluid-Particle System

The flow and particle properties of the system studied are listed in Table 1. It shows that the particles are spherical except for the smallest ones, 0.275 mm in diameter. The property of fluid is that of pure water at 23°C.

The spouted bed properties in a bed of height, H_m , in the condition of minimum spouting are given in Table 2. The spout diameter listed there is the mean diameter and ϵ_{ms} is the average void fraction based on H_m . The voidage everywhere in the spout is observed to be

Table 1. Properties of the water-glass particle system studied.
 $D_c = 101.6 \text{ mm}$; $D_H = 25.4 \text{ mm}$

d_p mm	ρ_p kg/m^3	u_{mf} mm/s	ε_{mf}	u_T^* m/s	ϕ_s expt
0.275	2570	0.85	0.390	0.0384	0.81
0.460	2480	2.44	0.401	0.0680	0.98
0.774	2470	5.18	0.407	0.115	1
0.995	2670	12.37	0.416	0.158	1

*calculated [14]

substantially less than unity and approximately uniform axially.

Correlation of Data and Comparison with Data for Coarse Particles

As can be seen in Figure 3, the minimum spouting velocity data are well correlated by the dimensionless form of the Mathur and Gishler relation [1] with u_{mf} replaced by $(u_{ms})_{Hm}$, that is

$$u_{ms}/(u_{ms})_{Hm} = h^{\frac{1}{2}}, \quad h > 1/4 \quad (1)$$

$(u_{ms})_{Hm}$ ranges from 17 to 29 % above u_{mf} due to wall (D_H for spouted bed is 15.52 mm and that for fluidized bed is 25.4 mm) and bed expansion effects. A modified Grbavcic et al.'s [8] equation, dotted line in Figure 3, gives predictions which are high. When $h < 1/4$

Table 2. Experimentally measured spouted bed properties in beds of height, H_m , in the condition of minimum spouting. $D_c = 50.8 \text{ mm}$; $D_H = 15.52 \text{ mm}$

d_p mm	d_t mm	H_m mm	D_s mm	$(u_{ms})_{Hm}$ $\text{m/s} \times 10^3$	$(\Delta P_{si})_{Hm}$ kN/m^2	$(\Delta P_s)_{Hm}$ kN/m^2	ε_{ms}
0.275	3.18	165.0	9.6	1.00	1.170	1.170	0.395
	6.35	158.0	9.9		1.068	1.096	0.402
	12.70	152.6	10.3		1.053	1.062	0.405
	19.05	146.0	11.1		1.033	1.037	0.404
0.460	3.18	133.4	11.5	2.93	0.905	0.905	0.408
	6.35	120.7	11.9		0.823	0.830	0.417
	12.70	104.0	13.0		0.711	0.717	0.416
	19.05	97.0	13.5		0.684	0.685	0.419
0.774	3.18	96.8	13.4	6.47	0.630	0.630	0.414
	6.35	90.5	13.8		0.595	0.599	0.420
	12.70	84.0	14.7		0.536	0.554	0.424
	19.05	82.1	15.3		0.533	0.543	0.424
0.995	3.18	83.7	15.0	16.00	0.677	0.667	0.433
	6.35	82.0	15.6		0.659	0.668	0.438
	12.70	77.8	16.2		0.624	0.629	0.435
	19.05	76.0	17.1		0.623	0.625	0.439

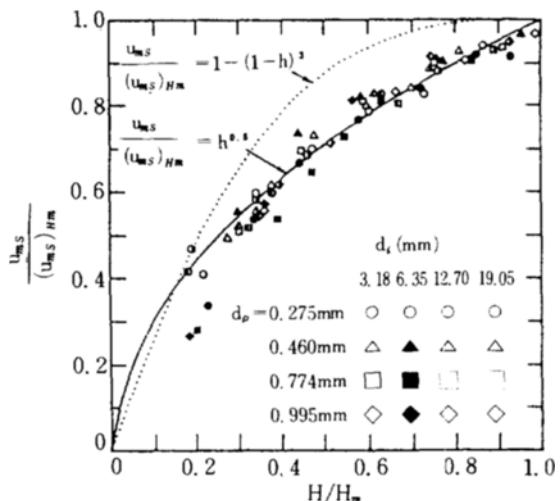


Fig. 3. Minimum spouting velocity-data and correlation.

neither of the equations fit the data well and predict significantly higher values for u_{ms} than found experimentally.

The effect of bed height on $\Delta P_{ms}/\Delta P_{mf}$ at minimum spouting (Figure 4) shows that the experimental values increase with h and reach an asymptotic value of about 0.85. This is considerably above the maxima predicted by the Mamuro and Hattori [5] and Lafroy and Davidson

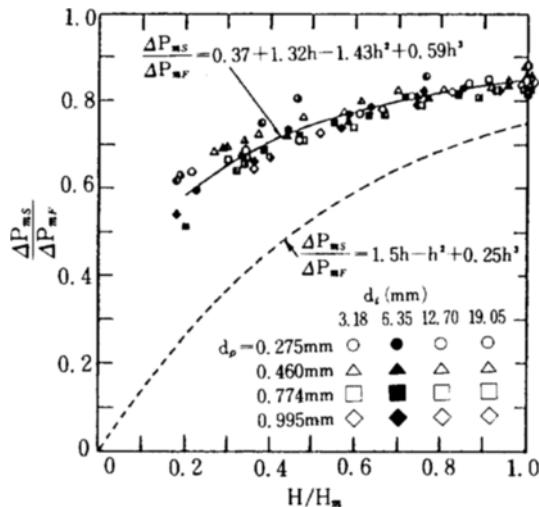


Fig. 4. Effect of bed height on the spouting pressure drop as $\Delta P_{ms}/\Delta P_{mf}$.

[6] theories (0.75 and 0.64 respectively).

The small particle data are well correlated by the equation

$$\Delta P_{ms}/P_{mf} = 0.37 + 1.32h - 1.43h^2 + 0.59h^3, \quad h > 0.2 \quad (2)$$

with correlation coefficient of 0.89. Based on the data for spouting of coarse particles with air and water, Grbavcic et al. [8] proposed the following equation.

$$\Delta P_{ms}/\Delta P_{mf} = 1.5h - h^2 + 0.25h^3 \quad (3)$$

As can be seen in Figure 4, Equation (3) predicts considerably lower values of $\Delta P_{ms}/(\Delta P_{mf})$ than found ex-

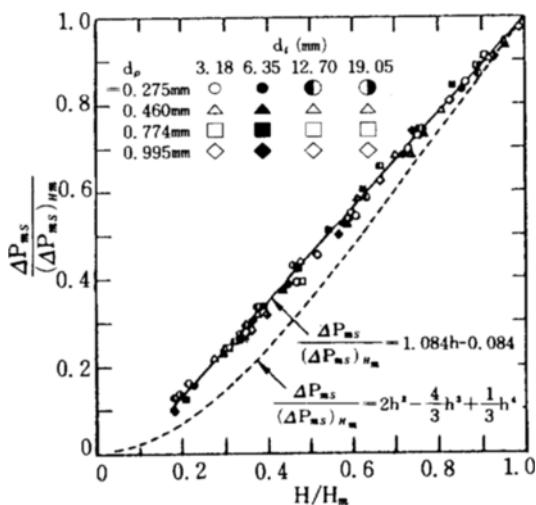


Fig. 5. Effect of bed height on the spouting pressure drop as $\Delta P_{ms}/(\Delta P_{ms})_{Hm}$.

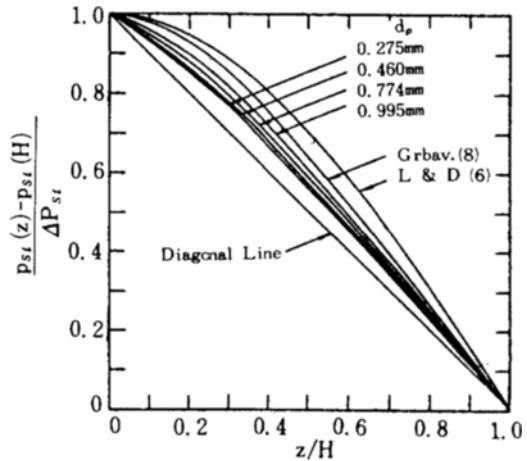


Fig. 6. Normalized spout-annulus interfacial pressure profile.

perimentally. The Morgan and Littman [11] theory predicts some of the high values of $\Delta P_{ms}/\Delta P_{mf}$ observed in experiments when d_i is large, but it is unsatisfactory in general.

The effect of h on the minimum spouting pressure drop ratio, $\Delta P_{ms}/(\Delta P_{ms})_{Hm}$, is also different between the small and large particles as shown in Figure 5. The small particle data correlate as

$$\Delta P_{ms}/(\Delta P_{ms})_{Hm} = 1.084h - 0.084, \quad h > 0.2 \quad (4)$$

with correlation coefficient of 0.99, while for coarse particles, the Grbavcic et al. [8] theory gives

$$\Delta P_{ms}/(\Delta P_{ms})_{Hm} = 2h^2 - (4/3)h^3 + (1/3)h^4 \quad (5)$$

Equation (5), which represents coarse particle data very well, predicts values for the ratio smaller than those for small particles.

The difference between the fine and coarse particle data is also indicated in the normalized spout-annulus interfacial pressure profile. The effect of d_i on the normalized interfacial pressure profile is very small for the small particle system in the minimum spouting condition and practically independent of d_i . However, the profile changes with d_p .

Least square fitting of the data to a third order polynomial gives

$$\frac{p_{st}(z) - p_{st}(H)}{\Delta P_{st}} = 1.0 - 0.6160(z/H) - 0.6573(z/H)^2 + 0.2733(z/H)^3 \quad \text{for } 0.275\text{mm particles,} \quad (6)$$

$$\frac{p_{st}(z) - p_{st}(H)}{\Delta P_{st}} = 1.0 - 0.5289(z/H) - 0.8045(z/H)^2 + 0.3343(z/H)^3 \quad \text{for } 0.460\text{mm particles,} \quad (7)$$

$$\frac{p_{st}(z) - p_{st}(H)}{\Delta P_{st}} =$$

$$1.0 - 0.4239(z/H) - 0.9850(z/H)^2 + 0.4089(z/H)^3$$

for 0.774mm particles (8), and

$$\frac{p_{st}(z) - p_{st}(H)}{\Delta P_{st}} =$$

$$1.0 - 0.2903(z/H) - 1.2182(z/H)^2 + 0.5085(z/H)^3$$

for 0.995mm particles. (9)

The correlation coefficients for Equations (6, 7, 8,) and (9) are 0.985, 0.983, 0.972 and 0.980, respectively. For each particle size, the deviations between the equation and data increase slightly as z/H approaches one.

Equation (6-9) are plotted in Figure 6 along with the profiles for coarse particles [6, 8]. As can be seen in Figure 6, the profile for the smallest particles, 0.275 mm in diameter, is close to linear profile which represents uniform fluidization. As the particle size increases, it goes toward the profiles for coarse particles [6, 8]. The normalized pressure profile in the spout is essentially the same as that at the interface for the small particles. These indicate substantial difference in the spout pressure profile between the fine and coarse particles which in turn have a substantial effect on the flow of fluid from the spout to the annulus and thereby on the annulus flow field.

Littman et al. [9] have derived a relationship H_m and D_s for coarse particles as

$$(H_m D_s) / (D_c^2 - D_s^2) = 0.345 (D_s / D_c)^{-0.384} \quad (10)$$

The small particle data follow this relationship quite well overall.

An equation for predicting H_m in terms of D_c , d_i and the fluid-particle properties is given by Littman et al. [10] as

$$(H_m d_i) / D_c^2 \equiv m = 0.218 + 0.005/A \quad (11)$$

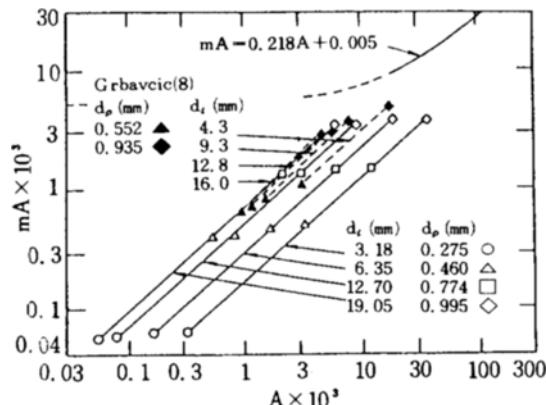


Fig. 7. Dimensionless maximum spoutable height relationship-effect of d_p and d_i .

Equation (11), which has been very successful in predicting H_m for coarse particles, is plotted in Figure 7 along with Grbavcic et al. [8] data. H_m in systems of small particles, drops off rapidly as A is reduced and it is clear that a different type of equation is necessary to describe H_m data in the low A regime ($A < 0.02$). Since ρ_p and ρ_f are essentially constant in this experiment, A is a function of d_p and d_i . H_m decreases as d_p and d_i increase and the effect of d_i on H_m is small compared to that of d_p . An empirical equation of the form $H_m = ad_p^b d_i^c$ was used to correlate the data and a least square fitting of the data gives

$$H_m = 98.34 d_i^{-0.07} d_p^{-0.50} - 7.76, \text{ mm} \quad (12)$$

with correlation coefficient of 0.97. Note that H_m is only a weak function of d_i as might be expected because of the low inlet jet momentum of the fluid.

The effect of d_i on H_m , however, increases considerably as the inertial force of the jet at minimum spouting increases and H_m varies as d_i^{-1} asymptotically for large particles [10]. The effect of particle size on H_m also changes with inlet jet momentum from $d_p^{0.5}$ to being independent of d_p . The effect of D_c on H_m is under investigation.

The spout diameter in the minimum spouting condition follows the equation

$$d_s(H, u_{ms}) / D_s(H_m, (u_{ms})_{H_m}) = 0.72h + 0.28 \quad (13)$$

as seen in Figure 8.

Littman et al.'s [10] correlation for coarse particles

$$D_s/d_i = [(2.10e^{-0.018/A} + 1.0) / 3.10]$$

$$[0.862 + 0.219(D_c/d_i) - 0.0053(D_c/d_i)^2] \quad (14)$$

does not predict the small particle data overall. The small particle data show that D_s/d_i increases linearly

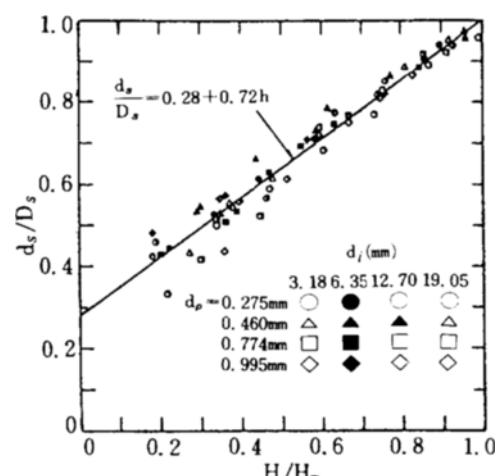


Fig. 8. Effect of bed height on the spout diameter in the minimum spouting condition.

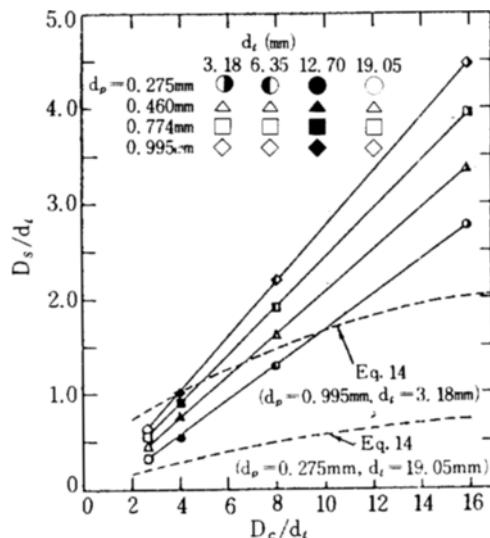


Fig. 9. Effect of D_s/d_i on D_s/d_i in the minimum spouting condition.

with D_s/d_i for each particle size and that the slope increases with particle size (Figure 9). For the largest spout inlet, smallest d_i/D_s , Equation (14) is in reasonably good agreement with experimental data. However, it under-predicts D_s/d_i as d_i increases for a given D_s .

In the absence of a general relationship for D_s , the best can be done if H_m is known to calculate D_s from Equation (10).

DISCUSSION

Equations representing the minimum spouting

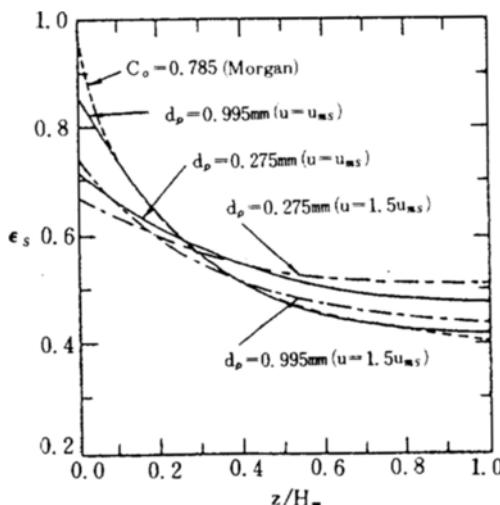


Fig. 10. Voids profiles in the spout.

characteristics of small particles do not in general correlate with those for coarse particles because the small particle data have been obtained in a different spouting regime. To understand this it is necessary to look into the stability conditions for spouted beds.

Morgan et al. [12] have calculated the spout voidage distribution at minimum spouting using a variational technique and the boundary conditions of $\epsilon_s(0) = 1$ and $\epsilon_s(H_m) = \epsilon_{mf}$. Their solution shows that no variational solution is obtained at minimum spouting for which ϵ decreases monotonically and satisfies the aforementioned boundary conditions unless a particular parameter, C_0 , is bounded by 0.215 and 0.785. Their axial voidage profile with C_0 equal to 0.785 (labeled Morgan) is given in Figure 10.

C_0 in the systems studied is essentially equal to $\Delta P_{ms}/P_{mf}$ and from the data in Figure 4 is clearly greater than 0.785. Furthermore, particles are observed in the inlet to the spout at the minimum spouting showing contrary to what is observed for coarse particles.

The voidage distribution in the spout in low inertia systems can be calculated very simply using only the spout pressure profile as is apparent once the momentum equation is simplified to represent this case. For the fluid and particle flow through an element of the spout assuming a constant spout diameter and radially averaged velocities, the pressure and voidage

$$\frac{d}{dz}(\epsilon_s u_s^2) + \rho_f \frac{d}{dz}[(1-\epsilon_s)v_s^2] = -\frac{dp_s}{dz} - \rho_f g - (1-\epsilon_s)(\rho_f - \rho_s)g$$

In this work, the inertial term is small at minimum spouting. Neglecting the terms on the left hand side of equation (15),

$$-\left[\frac{dp_s}{dz} + \rho_f g\right] = (1-\epsilon_s)(\rho_f - \rho_s)g$$

which shows the simple relationship between P_s and ϵ_s .

Using the experimental axial pressure profile in the spout at minimum spouting in a bed of height H_m , ϵ_s was calculated from equation (16) for the particle sizes 0.275 and 0.995 mm. The results are shown in Figure 10. In all cases, $\epsilon_s(0) < 1$. The profiles for 0.460 and 0.774 mm particles are in between those for 0.275 and 0.995 mm particles and the particle size raises the value of $\epsilon_s(0)$ toward asymptotic value of $\epsilon_s(0) = 1$ which is the characteristic of coarse particle system. Thus, the regime transition from the coarse to fine particle spouting is defined as occurring when $\epsilon_s(0)$ goes from 1 to less than 1 with the simultaneous rise in C_0 above 0.785. Support to this view lies in the fact that $d\epsilon_s(0)/dz$ is infinite when C_0 equals to 0.785 (see Morgan in Figure 10).

The voidage profiles with $u/u_{ms} = 1.5$ were calculated using equation (16) and the experimental

pressure profile. As seen in Figure 9, they are flatter than those at minimum spouting as expected.

The spout voidage distributions also tell a great deal about the particle circulation patterns. Whenever the profiles are as linear as they are generally in the top portion of the spout, equation (16) shows that the spout voidage is constant. It has been observed that particle velocities in the top portion of the bed do not vary significantly with z indicating that there is very little particle circulation there from the annulus into the spout. In general, it is observed that the particles enter the spout primarily in the lower part of the bed. The particle residence time distribution in the annulus is, therefore, much narrower than when using coarse particles. Lowering the particle diameter or increasing the inlet fluid velocity above the minimum both narrow the particle residence time distribution in the annulus in the direction of what would occur by inserting a draft tube in the bed.

As the column diameter of the spouted in this study is relatively small compared to those applied for coarse particle system, the effect of D_c on the basic spouting parameters is under investigation. The flow characteristics of fluid and particles in the annulus of small particle beds, which are substantially different from those for coarse particles, will be discussed in another paper.

CONCLUSIONS

1. The basic spouting parameters such as the bed pressure drop, interfacial pressure profile, maximum spoutable height and spout diameter at minimum spouting do not correlate with data for coarse particles implying a new spouting regime.

2. Stability considerations show that the transition from the coarse to fine particle spouting regime may be defined as occurring when $\epsilon_s(0)$ goes from 1 to less than 1.

3. The voidage in the spout is observed to be more uniform axially than in coarse particle systems.

ACKNOWLEDGEMENT

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NOMENCLATURE

A	$[\rho_p / (\rho_p - \rho_f)] [u_{mf} u_t / g d_t]$
C_0	$\frac{\int_0^1 1 - \epsilon_s \left(\frac{z}{H} \right)}{1 - \epsilon_{mf}} d \left(\frac{z}{H} \right)$
d_t	Spout inlet tube diameter

d_p	: Particle diameter
d_s	: Spout diameter
D_c	: Column diameter
D_h	: Hydraulic diameter
D_s	: d_s in a bed of height H_m in the minimum spouting condition
g	: Gravitational acceleration
h	: H/H_m
H	: Bed height
H_p	: Jet penetration or spout fountain height
H_m	: Maximum spoutable height
m	: $(H_m d_i) / D_c^2$
p_s	: Fluid pressure in the spout
ΔP_{ms}	: Minimum fluidization pressure drop in a bed of height, H
ΔP_{ms}	: P_s in the minimum spouting condition
$(\Delta P_{ms})_{H_m}$: ΔP_{ms} at the bed of height, H_m
ΔP_s	: $P_s(0) - P_s(0)$
P_{s1}	: Pressure at the spout-annulus interface
u_{mf}	: Minimum fluidization velocity
u_{ms}	: Minimum spouting velocity
u_s	: Interstitial fluid velocity in the spout
u_t	: Terminal velocity of the particle
V_{mf}	: Flowrate at minimum fluidization
V_{ms}	: Flowrate at minimum spouting
v_s	: Interstitial particle velocity in the spout
z	: Vertical coordinate measured from spout inlet
ϵ	: Bed voidage
ϵ_{mf}	: Voids at minimum fluidization condition
ϵ_{ms}	: Voids at minimum spouting condition
ϵ_s	: Spout voidage
μ	: Fluid viscosity
ρ_f	: Fluid density
ρ_p	: Particle density
ϕ_s	: Particle shape factor

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